

Lecture 6: Mathematics physics.

- linear algebra applied to function spaces

Positive definite: $\bar{V}^T A \bar{V} \geq 0 \quad \forall \bar{V} \neq 0$

$\det A \geq 0$ - Mechanics systems

- Physically, what systems are represented using positive definite matrices and why do these systems need to be positive definite?

• positive definite symmetric matrices

orthogonal + symmetric $\rightarrow$ analog of unitary

Symmetric $\rightarrow$ analog of Hermitian

A matrix is diagonalizable iff the matrix is positive definite and symmetric.

- What properties define functions that live in an space? Is it the basis?

Ston + Goldbart $\Rightarrow$ chapters 2 & 4

• $F[a,b] \rightarrow L^p[a,b] \rightarrow L^2[a,b]$

General function space $\rightarrow$ norm (not necessarily finite complete basis) $\rightarrow$ complete expansion $\rightarrow$ Complete basis: functions live in same space

$$d(f,g) = \|f-g\|_p$$

- positive definite inner product

- What are Sturm-Liouville problems?

El $\Rightarrow$ function space with positive definite inner product.

- Functions must have the same domain for the purposes of our course

$C^n \rightarrow$ n-th times differentiable. Subspace of functions often show physical systems.

- Operators might operate in a subspace.

- Is there a basis in a subspace that is not a basis of general state.

Fast time

$$f(x) = \sum_{n=1}^{\infty} a_n \phi_n(x) = \sum_{mn} a_n |\phi_m\rangle \langle \phi_n| \phi_m\rangle = \sum_{mn} a_n U_{nm} |\phi_m\rangle$$

$$= \sum_m a_m \phi_m(x) \quad \text{if our states are orthonormal}$$

$$\sum_m a_m = \sum_{mn} a_n U_{nm}$$

What is the reason for this? $\hat{U} = \int_{-\infty}^{\infty} \delta(x-y) dx$

Box transformation are unitary

Prove this now. This has one dimension

$$b_n = \sum_m U_{nm} a_m \quad a_n = \sum_{m'} U_{nm'} b_{m'}$$

Inverse, transpose, come together.

$$\langle f | A | g \rangle = \langle A f | g \rangle \quad \text{iff } A^\dagger = A$$

$$A_{mn} = \langle \phi_m | A | \phi_n \rangle$$

$$= (A)_{mn} \quad (A_{mn})^{\dagger T} = (A^\dagger)_{nm}$$

$$A_{m'n'} = \sum_m U_{nm} A_{mn} (U_{n'm'})^{\dagger T} \quad A' = U A U^\dagger \rightarrow \text{Similar transform}$$

Eigenvalue problems

Self

Hermitian $\Leftrightarrow$ Adjoint same time

$$Hf = \lambda f \quad H \Rightarrow \text{complete basis} \quad \text{and} \quad H = H^\dagger$$

• Spectral theorem

• H and $H^\dagger$ are assumed to be self-adjoint.

• Does some domain imply complete basis? what about boundary conditions?

• A, B have the same eigenvalues iff $[A, B] = 0$

See Weinberg books on Quantum Mechanics

Differential Equations

Know how to solve needs once they have form

Ans given - Act as minimum

+ Linear functions have the growth as are interested in solving to 1st power

$$h_y = 0$$

$\rightarrow$ guess

$\uparrow$

linear operator

+ ODES $\Rightarrow$ Separation of variables to solve PDES live in x^2

PDEs reduce to Systems of ODEs

+ Kuramoto - Shroeder system

+ Burgers

$$h_y = 0 \Rightarrow \text{Homogeneous} \rightarrow \text{chapter 7}$$

$$h_y = \lambda y \Rightarrow \text{eigenvalue} \rightarrow \text{chapter 8}$$

$$h_y = f(x) \Rightarrow \text{inhomogeneous (forcing term)} \Rightarrow \text{Green function method (chapter 10)}$$

$$p_0(x) \frac{d^n y}{dx^n} + p_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + p_{n-1}(x) y = f(x)$$

+ Using derivatives in order 2

+ Unstable solutions for eigen of order above 2

Constant coefficients:

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_n y' = 0$$

The answer is $y = e^{mx} \Rightarrow$ constant coefficients

$$(a_0 m^n + a_1 m^{n-1} + \dots + a_n) = 0 \quad \text{polynomial} \quad n \text{ solutions}$$

• System of equations with constant coefficients

$$\dot{y} = A y \quad y(t) = e^{At} y_0$$

How to turn higher order into first order

$$\ddot{x}_1 = -\omega^2 x_1 \Rightarrow \text{oscillator eqn} \quad 2^{\text{nd}} \text{ order into two first order}$$

$$\text{Equilibrium pos occur when } \frac{dV_L}{dt} = \frac{dV_U}{dt}$$

What's an unstable equilibrium point?

Next time segment of v